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FORMULATION OF A CONSISTENT MACROECONOMIC MODEL WITH THE BOND MARKET: SIMPLE ANALYTICS

By Akpan h. Ekpo ¹

Abstract

The paper has discussed the traditional macroeconomic model and provides a simple analysis of the bond market as a proxy for a rudimentary treatment of the financial market. An analysis of the bond market with a simplified assumption still ensures consistency in the formulation of a macroeconomic model in a static context. Consequently, the efforts of economies in the West African sub-region to establish capital market is a step in the right direction.

Keywords: Bond market, West Africa

JEL Classification: C5, C51, D1, D11, D21, D4, D53

I. INTRODUCTION

The keynesian 'revolution' encourages economists to build models that reflect the aggregate economy. The objectives of such models are generally to capture the realities of an economy as it struggles to provide full-employment, price stability, growth and stability in the external sector. The simple macroeconomic model includes the output/product market, money market, labour market, and the external sector to capture the openness of the economy. The money market is often assumed in traditional macroeconomic model to capture the financial market. At best, Walras law is generally invoked to suppress the financial and other relevant markets.¹

In the advanced economies, given the sophistication of their financial markets, several macroeconomic models incorporate the complexities of the financial markets. In other words, the models with the financial realities reject the traditional macroeconomic model in assuming away the financial market. It is important to state that even within the traditional and simple analysis, Patinkin (1964) raises the matter of considering the financial market through the bond market if the macroeconomic framework has to be consistent.

There is also a debate as to whether financial stability includes macroeconomic stability or whether the reverse is the case. Nonetheless, the global economic crisis

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of 2008 on account of which the world is still experiencing sluggish growth has re-echoed the importance of making explicit the financial market in macroeconomic analysis.

The West African sub-region's financial market is still emerging. At present, there are stock exchanges in Nigeria, Ghana, Cote'Ivoire, and Senegal. The Gambia and Sierra-Leone have been making preparations towards establishing stock exchanges. The main idea is to raise capital especially within the private sector to finance long-term development.

The objective of this paper is to highlight theoretically the importance of the financial market using the bond market as a proxy in formulating a consistent traditional/conventional macroeconomic model. Following the introduction, section 2 briefly presents the traditional macroeconomic framework. Section 3 examines the bond market. Section 4 concludes the paper. It is expected that the simple analytics would drive home the significance of the bond market in raising capital in the economies of the West African sub-region.

2. TRADITIONAL MACROECONOMIC MODEL

In principles of economics, macroeconomic equilibrium occurs at the level of income and output where $Y = C + I + G$. $Y = C + I + G + X - M$ is a necessary but not a sufficient condition for commodity market equilibrium. Macroeconomic equilibrium requires that all markets be in equilibrium.

In addition, the $Y = C + I + G + X - M$ condition gives a unique equilibrium income only in the simplest model which ignores prices, interest rate and the effect of wealth on the demand for commodities. The $X - M$ component represents an open economy. However, in this analysis, the paper assumes a closed economy.

Y = income; I = investment spending; G = government spending. In order to provide a framework for understanding macroeconomic model, The paper examines the rudiments of the basic macroeconomic framework:

2.1 The Output Market

The basic assumption underlying the aggregate level of analysis in the paper is that the economy produces only one commodity. This product can be used either for consumption or it can be invested. When invested, it is accumulated as a capital good and combined with the other factors of production (say labour) to produce more output. Assuming a closed economy, equilibrium in the commodity market is shown by:

$$Y = C + I + G \dots \dots \dots (1)$$

Where Y = real output or national income

C = real private demand for consumption

I = real private demand for investment

G = real government expenditure

Equation (1) must not be confused with the national income identity. Equation (1) is an equilibrium condition which may not be satisfied, if total desired expenditure ($C + I + G$) is also less than total income (Y), then ($C + I + G$) is less than output. This cannot be an equilibrium situation, that is, output is less than the demand for commodities. Something must change; either demand to fall or supply to rise or both.

Also, if $C + I + G > Y$, then people desire to spend more than total income and output. This is possible only by borrowing, drawing on savings or printing money) consequently, output must rise or demand must fall or both.

Therefore, $Y = C + I + G$ is a necessary condition for equilibrium in the commodity or product market. There are many different income levels for which $Y = C + I + G$, one for each interest rate and price level.

Note that equation (1) is written in real terms. It will not alter the analysis if nominal values are used. Real terms have been used to capture the essence of inflation. Though in some markets, it is controversial whether to use real or nominal values due to the notion of "**money illusion**"

It is important to ascertain the determinants of (C, I, G). The simplest theory postulates the consumption function as:

$$C = C(Y) \dots \dots \dots (2)$$

$$0 < \frac{\partial C}{\partial Y} < 1 \dots \dots \dots (3)$$

With I and G remaining constant ($I = I_0, G = G_0$)

Equation (2) says that consumption depends upon income while investment and government expenditure are both determinant exogenously.

Substituting these relationship into equation (1) the paper has:

$$Y = C(Y) + I_0 + G_0 \dots \dots \dots (4)$$

So that Y can be solved uniquely in terms of the exogenous values of investment and government expenditures.

Let consumption be a function of disposable income: That is:

$$C = C(yd); 0 < \frac{\partial C}{\partial yd} < \dots\dots\dots(5)$$

Where $Yd = Y - T$

Yd = real disposable income;

T = taxes;

T can be endogenous: $T_o = T_o + tY$

If we assume T to be constant (T_o) then we have a Keynesian short-run consumption function:

$$C = c(Y-T)$$

However, after Keynes, there has emerged sophisticated postulations of the consumption function, namely;

- Shifting income hypothesis.
- Relative income hypothesis.
- Permanent income hypothesis, and
- Life cycle income hypothesis.

Most of these postulations have been derived from utility maximization. The outcome of these studies has modified the typical Keynesian SR consumption function.

The paper cannot reconcile the on-going controversies as regards the appropriate consumption function. However, the up-shot of these contributions is that consumption is, however, made to depend on wealth, disposable income and interest rate.

The latter is not completely accepted by most authors. The Paper can thus state a consumption function as:

$$C = C(yd, r, Z/p) \dots\dots\dots(6)$$

$$0 < \frac{\partial C}{\partial Y} < 1; \frac{\partial C}{\partial Z/p} > 0; \frac{\partial C}{\partial r} \geq 0$$

Where r = rate of interest.

z/p = real wealth of the private sector.

For now, drop r in order to have:

$$C = C(Y_d, z/p) \dots\dots\dots(7)$$

Equations (6) or (7) can be described as a long-run consumption function incorporating wealth as a proxy for all the other determinants of consumption. The investment function can be postulated in its simplest form as:

$$I = I(r) \quad I' < 0 \dots\dots\dots(8)$$

Equation (8) holds based on the discounted present value criterion. A profit maximizing firm should invest on those projects yielding a positive discounted present value.

As the interest rate rises, the number of projects having this property will fall so that the amount of investment undertaken by the firm will fall, thus justifying equation (8)

The investment function can also be written as:

$$I = I(r, Y, K) \dots\dots\dots(9)$$

$$\frac{\partial I}{\partial r} < 0, \frac{\partial I}{\partial y} > 0; \frac{\partial I}{\partial k} < 0$$

Equation (8) shows that investment depends positively on the level of income; negatively on the interest rate and level of existing capital stock K .

Substituting equations (6) and (8) into the product market equilibrium (equation 1) yields the **IS** curve

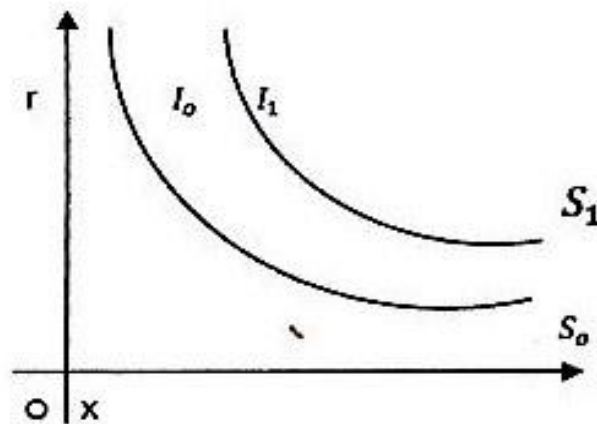
$$Y = C(Y - T) + I(r) + G \dots\dots\dots(10)$$

The IS curve gives the combination of Y and r which keep the product or commodity market in equilibrium. Differentiating both sides of (9) with respect to Y , the paper has:

$$\left. \frac{dr}{dy} \right|_{IS} = \frac{1-c^1}{I^1} < 0 \dots\dots\dots(11)$$

Implying a downward slopping IS curve

Fig. 1: The IS Curve



The IS curve is downward sloping because as income rises, consumption rises but less than income. Invariably, the only way the excess output can be absorbed is by additional investment and for this to occur, the interest rate must fall.

It should be clear that the position of the IS curve depends on the position of the investment function, consumption function and government expenditure. An increase in government expenditure, or an outward shift in either C or I function will push the IS curve downward as IS_0 (see the appendix for an algebraic derivation of the IS equation).

2.2 The Money Market

Combining the quantity theory of exchange, the micro-foundations of monetary theory as well as the Keynesian liquidity preference theory, the paper can state the demand for money thus:

$$L/P = L(Y, r) \dots \dots \dots (12)$$

Where $Y' > 0$; $r' < 0$

The real money supply is determined exogenously through the Central Bank:

$$M/P = M_0/p \dots \dots \dots (13)$$

It should be noted that the money supply can be determined endogenously through the commercial banking system and the open market operations of the Central Bank.

Setting equation (12) equal to equation (13) will yield equilibrium in the money market in a static model. Solving the equilibrium for either r or Y will result in an LM

equation. The LM equation gives the combinations of r and Y that clears the money market in a static model. An algebraic derivation of the LM equation is shown in the appendix.

2.3 Equilibrium In The Product/Output and Money Markets

The equilibrium levels of income and interest rate is arrived at by combining the IS and LM curves/equations. Rewriting the two equations:~

$$Y - C(Y - T) - I(r) - G = 0 \quad \dots\dots\dots(14)$$

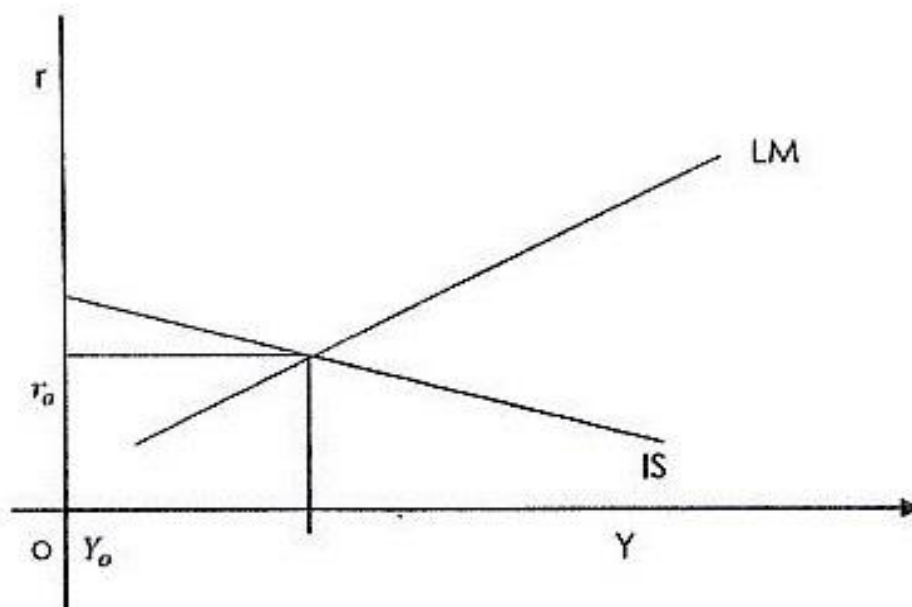
$$\frac{m}{p} - L(Y, r) = 0 \quad \dots\dots\dots(15)$$

*Note that taxes T is exogenously determined.

It is straight forward that based on our assumptions, equations (14) and (15) can be solved uniquely for Y , r in terms of P , G , M , T .

The graphical solution is given below

Figure 2: Determination of Equilibrium in the Output and Money markets



The solution:

$$Y = Y(P; G, M, T) \dots\dots\dots (16)$$

Which relates equilibrium income as determined by demand to price. It can thus be seen as an aggregate demand function.

Differentiating equations (14) and (15) wrt P we have:

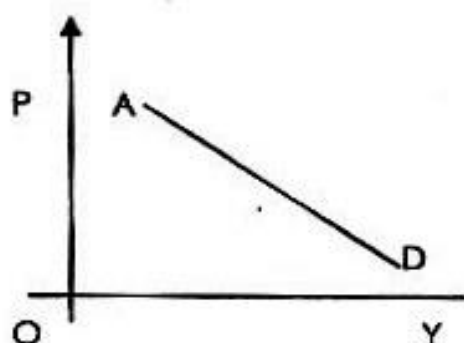
$$\begin{pmatrix} 1 - \frac{\partial c}{\partial y} & -\frac{\partial I}{\partial r} \\ -\frac{\partial L/P}{\partial y} & -\frac{\partial L/P}{\partial r} \end{pmatrix} \begin{pmatrix} \frac{\partial Y}{\partial P} \\ \frac{\partial r}{\partial P} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{m}{P^2} \end{pmatrix}$$

and solving for $\frac{\partial Y}{\partial P}$ via scalar expansion

$$\frac{\partial Y}{\partial P} = \frac{(\frac{\partial I}{\partial r}) m/P^2}{-\frac{\partial L/P}{\partial r} (\frac{1-\partial I}{\partial Y}) - (\frac{\partial L/P}{\partial Y}) (\frac{\partial I}{\partial r})} < 0 \dots\dots\dots(17)$$

Thus, the AD Curve is downward sloping:

Fig. 3: THE AGGREGATE DEMAND CURVE



In the traditional model, the eclectic aggregate supply curve can be derived through the labour market (see Miller, 1995).

The money market in the traditional model subsumes the financial wealth. The discussion of the long-run consumption function which incorporates the wealth

effect does not explicitly consider the financial market. The paper next provide a simple analysis of the bond/financial market for some consistency within the basic model.

3.0 THE BOND MARKET

Traditionally, the bond market is often assumed away in the macro-model. This is often allowed by invoking Walras' law. According to this law, if all markets are in equilibrium but one market then that one market is indeed in equilibrium. As part of building a consistent macro-model, it is necessary to consider the bond market because if it is not in equilibrium, the system may in theory and practice exhibit some problems.

What is presented below is an introduction and simplified version to ensure consistency with the traditional macro-model. The paper notes that the issue of formulating a consistent macro-model is much more involved; it concerns analysing the budget constraints of all economic agents and that of the entire economy as well as the underlying various behavioural relationships. A consistent framework makes explicit the several implicit assumptions of the traditional IS – LM framework.

In the macro-model, a bond is a general type of nonmonetary financial asset issued by firms to finance investment and by the government to help finance government spending. It will be generally assumed that each bond pays 1*N* in interest each period and has no maturity date. In addition, both firms and government supply bonds, households (which hold the bonds) do not distinguish between government and private bonds. Let us state the following equations:

3.1 Demand for Bonds:

$$\bar{\beta}^d = \frac{i\beta^d}{p} = \beta_o^d + ey + qr + h\frac{m}{p} \dots\dots\dots(18)$$

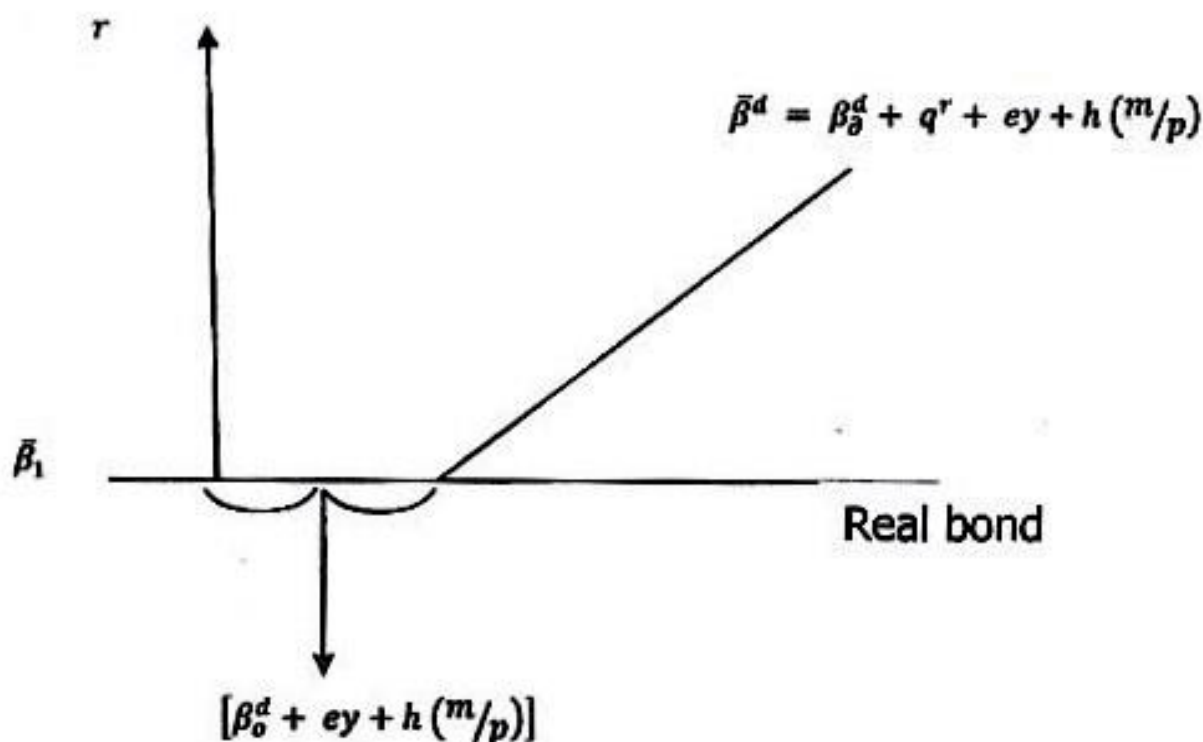
$$\beta^d = \beta(y, r, m/p) \text{ (general form)}$$

Equation (18) is the demand for bond equation, $\bar{\beta}^d$ is the number of bonds demanded. Each bond pays 1*N* per period in interest thus $1/r$ is the price of bond. All households demand an amount of real bonds $\left(\frac{1/r\beta_d}{p}\right)$. Equation (18)

assumes that real bonds demanded: (1) rises with income $\frac{\partial \bar{\beta}^d}{\partial y} = e > 0$; (2) increases as the rate of interest rises $\left[\frac{\partial \bar{\beta}^d}{\partial r} = g > 0\right]$; and (3) rises as real cash

balances grow $[\frac{\partial \bar{\beta}^d}{\partial \mu \gamma \nu} = h > 0]$. The demand for real bonds has a positive slope $[\frac{\partial \bar{\beta}^d}{\partial r} > 0]$. Its horizontal intercept is found by setting $r = 0$. The curve shifts as income or real cash balances or the constant term changes.

Fig. 4: DEMAND FOR BONDS



$$\bar{\beta}_0 = \frac{1}{r} \beta_0^d = (1/r) \frac{\beta_0^d}{p} \Rightarrow \text{govt. bonds}$$

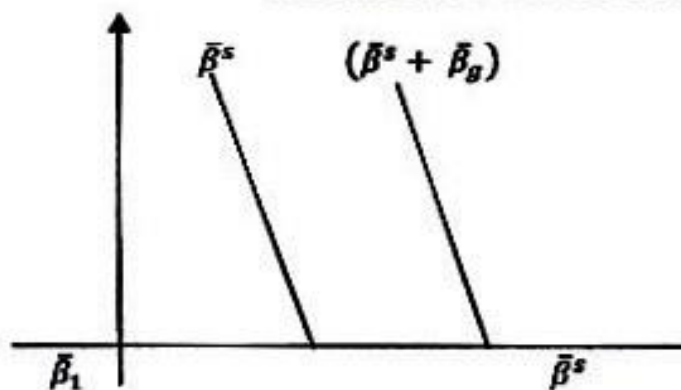
3.2 Bond Supply

$$\bar{\beta}^s = \frac{1}{r} \beta^s = \beta_0^s - J_r + s y \dots \dots \dots (19)$$

$$\bar{\beta}^s = \beta^s (r, y) \text{ (general form)}$$

Equation (19) is the bond supply equation. It depends on a constant term, on the interest rate and on income. Firms desire to invest more if the interest rate falls and to finance such investment by selling more bonds. Increases in income shift $\bar{\beta}_s$ to the right; low values of income shift $\bar{\beta}_s$ to the left.

FIGURE 5: SUPPLY OF BONDS

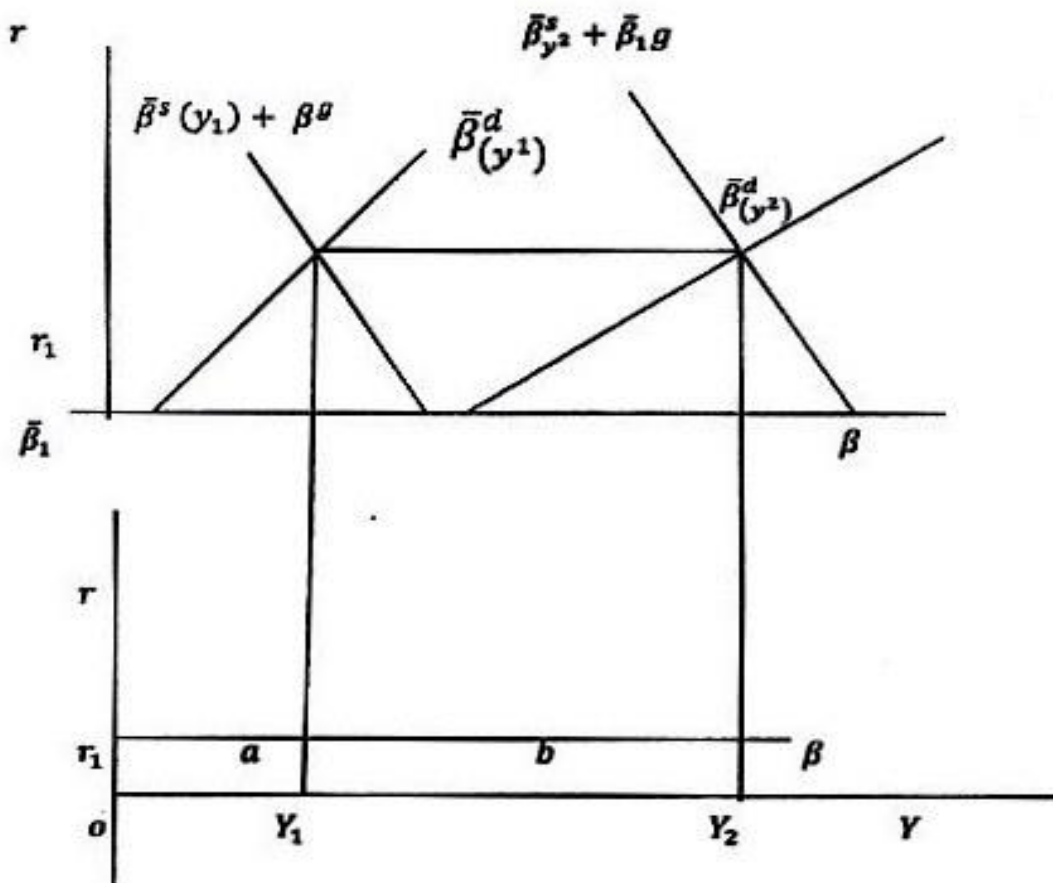


The total supply of bonds is the sum of bonds from firms and government bonds. An increase in any will shift the supply of bonds to the right.

Bond Market Equilibrium

$$\bar{\beta}^d = \bar{\beta}^s + \bar{\beta}_g \dots\dots\dots (20)$$

Fig. 6: Graphical Derivation of the $\beta\beta$ Curve

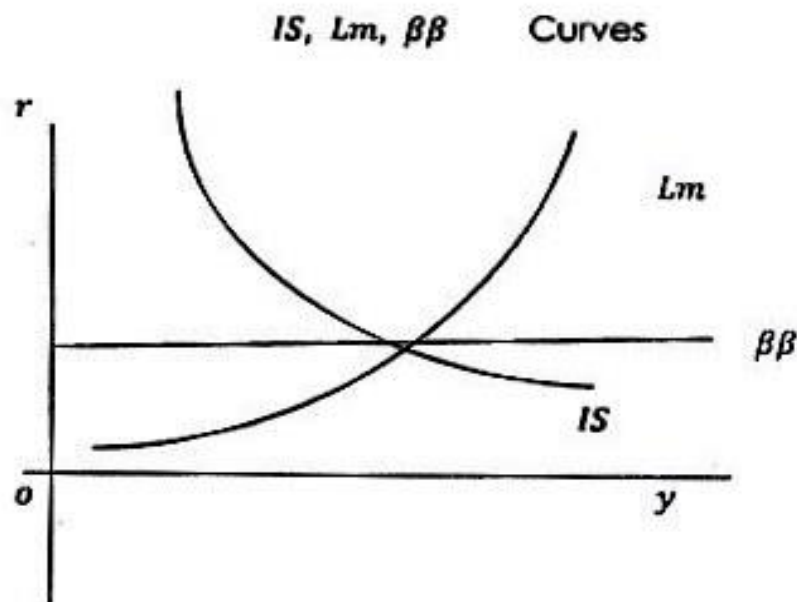


Bond market equilibrium is found at the point where $\bar{\beta}^d = \bar{\beta}^s + \bar{\beta}_g$. The initial interest rate needed to clear the bond market is r_1 . If the Walrasian auctioneer calls out a higher income (Y_2), the increase will be both bond demand and bond supply. $\bar{\beta}^d$ and $\bar{\beta}^s$ shift to the right by the same amount when income rises. This assumption of equal slopes is made just for simplicity.

$$\left[\frac{\partial \bar{\beta}^d}{\partial y} = e; \frac{\partial \bar{\beta}^s}{\partial y} = s. \text{ Hence } e = s \right]$$

Plotting points a and b on to the lower graph results in an horizontal $\beta\beta$ curve which shows the combinations of r and y that clear the bond market. It is anticipated that if the bond market is incorporated into the traditional IS-LM, the system can be graphed thus:

Fig. 7: OUTPUT, MONEY AND BOND MARKET IN EQUILIBRIUM



Note that if the $\beta\beta$ curve does not clear the traditional model, there will be several equilibrium points. Note also that the horizontal $\beta\beta$ curve is due to our simplified assumption (see the appendix for algebraic derivation of the BB curve).

4. CONCLUSION

The paper has summarized the traditional macroeconomic model with an analysis of a simple bond market as a proxy for the financial market. If the bond market is not considered explicitly, there could be several equilibrium points in a static model. Under such a scenario, it would be difficult to ascertain which combination of income and the rate of interest that clears both the output and money market. A consideration of the bond market with a simplified assumption still ensures macroeconomic stability within a static partial equilibrium analysis.

It is a truism that although macroeconomic management in developing countries has received increased attention the analytical framework for the study of macroeconomic issues seems to have given scant attention to the role of bond markets. Instead there is a proliferation of models with different and often conflicting assumptions about degree of capital mobility, the functioning of exchange rate regime, expectations, degree of wage – price flexibility, role of dynamic responses to policy and exogenous shocks, etc. The new generation of macroeconomic models that stresses those assumptions are a case in point. This study is an attempt to extend the theoretical frontiers of textbook rendition of the traditional macroeconomic model of a developing economy by incorporating the bond market. Thus consistency is achieved for the macroeconomic framework. It has underpinned the wisdom in developing capital markets by developing countries including the ECOWAS sub-region.

Even so, the effort does not necessarily represent the last word. Model building is an evolutionary process that will continue to reflect new developments in economic theory, better understanding of the structure of developing countries etc.

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APPENDIX

Algebraically an IS curve which incorporates wealth (Z) can be derived thus:

Assume:

$$C = a + b_1 (Y - T) + b_2 (Z/p) \dots\dots\dots(A1)$$

$$I = I_0 - Vr \dots\dots\dots(A2)$$

$$G = G_0 \dots\dots\dots(A3)$$

$Z/p =$ the real value of wealth

a, b, c, l, and V are parameters, all of which > 0

Substituting (13'), (13'') and 13''') into equation (5) we have

$$Y = a + b_1(Y - T) + b_2 C(Z/p) + I_0 - Vr + G_0$$

$$r = \frac{a + I_0 - G_0 - b_1 T + b_2 Z/p}{V} - Y \frac{(1 - b_1)}{V} \dots\dots\dots(A4)$$

Equation (14) is the IS curve and it incorporates the wealth variable

This IS curve slopes downwards:

$$\left. \frac{dr}{dy} \right|_{IS} = - \frac{(1 - b_1)}{V}$$

Deriving the LM equation in a closed system.

$$L/p = L_0 + ky - qr \dots\dots\dots(B1)$$

$$\frac{M^s}{p} = \frac{m}{p} \text{ (Money supply)} \dots\dots\dots(B2)$$

L = demand for liquidity preference

P = price level

r = interest rate

Y = income

M^s = money supply

$$\text{At equilibrium: } L_0 + Ky - qr = \frac{m}{p} \quad (B3)$$

Solve for r:

$$r = \frac{(L_0 - m/p)}{q} + \frac{k}{q} y \quad \text{LM equation} \dots\dots\dots(B4)$$

The term on the rhs gives the value for r that clears the money market when income is 0. The term $(L_0 - m/p)/q$ is the vertical intercept of the LM curve. This intercept could be positive or negative depending on the magnitude of L_0 relative to m/p . The intercept will increase and LM will shift up and to the left for any increase in L_0 or decrease in $(m/p \text{ or } q)$.

The shape of the LM curve is the coefficient of Y in the LM equation since

$$\left. \frac{dr}{dy} \right|_{\text{LM}} = \frac{k}{q} > 0$$

Any movement along an LM requires r and y to change in a way that will keep L/p constant since m/p is fixed and L/p must equal m/p .

The change in L/p as we move up LM is composed of two different effects. First, as the interest rate rises, L/p declines by $(-q) \times (dr)$.

Second, as income rises, L/p increases by the amount (K) times $(dy) \Rightarrow [k \times dy]$. Since the next change in L/p must be zero, it follows that $(-q) dr + (k) dy = 0$ (B5)

$$\left. \frac{dr}{dy} \right|_{\text{LM}} = \frac{k}{q} = \text{slope of the LM equation.}$$

The magnitude of "q" affects the slope of the LM. As "q" gets smaller, the shape $\frac{k}{q}$ grows larger. In the limit, as q approaches zero, the term $\frac{k}{q}$ approaches infinity. This leads to a vertical LM curve on the usual classical case.

On the other hand, if L/p responds to interest rate changes very strongly then "q" is very large and the slope of LM (k/q) becomes small. In the limit, as "q"

approaches infinity, the slope of LM approaches zero. That is, $dr/dy = 0$ which corresponds to a horizontal LM curve. For many years, Keynesian economists thought that the LM curve looked like this during the great depression period of the 1930s.

Algebraic Derivation of the $\beta\beta$ Curve

Setting equation (18) equals (19) we have:

$$\beta_0^d + ey + qr + h(m/p) = (\beta_0^s - jr + sy) + \bar{\beta}_g \quad (C1)$$

Solving for r :

$$r = \left[\frac{\beta_0^d + h(m/p) - \beta_0^s - \bar{\beta}_g}{g+j} \right] - \frac{(e-s)y}{g+j} \dots\dots\dots(C2)$$

Equation (4) is the $\beta\beta$ curve. If $y = 0$, then

$$r = - \frac{\beta_0^d - h(m/p) + \beta_0^s + \beta^g}{g+j} \dots\dots\dots(C3)$$

which is the intercept of the $\beta\beta$ curve. Since we assumed that $(e = s)$, the coefficient of the y term in the equilibrium equation becomes zero. The $\beta\beta$ curve has a zero slope $\frac{\partial r}{\partial y} = - \frac{(e-s)}{g+j} = 0$.

This reduces the $\beta\beta$ equation to:

$$r = \left[\frac{\beta_0^d + h(m/p)}{g+j} \right] - \frac{\beta_0^s + \beta^g}{g+j} \dots\dots\dots(C4)$$

Since $(g + j) > 0$, we get a positive equilibrium interest rate (r) only if the numerator of the bracketed term on the rhs of equation (C4) is negative .

Shifts in the $\beta\beta$ Curve

The r which clears the bond market will increase and $\beta\beta$ will shift upward if $\beta_0^s + \beta^g$ increases, β_0^d falls and m/p falls either via a higher p or lower m .

An exogenous increase in bond supply ($\beta_0^s + \beta_1^g$) would shift the original bond supply curve to the right and give a higher equilibrium r .